# Student Challenges Integrating Math and Physics with Data Analysis

By

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MEET 2004
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# Acknowledgements

I wouldn't have been able to do this research without the support of many people especially my beautiful wife, Mei, and my exceptionally generous and thoughtful boss, Tim Erickson.  I would also like to thank the students involved in the research for tolerating my questions, for trying their best on the difficult labs, and for their thoughtful feedback.  And I would like to thank all the teachers involved in the project who let me invade their classrooms and observe their students, especially the two teachers at Oceanbeach High and Eastwood High.

# Problem Statement

There are important gaps in our math and science education standards. In the gaps between the standards of math and physics, critical abilities and skills are being left behind. For example, real data is not traditionally discussed in math classes along with critical issues surrounding measurement, units, and variability. In Paul Foerster's book (1999), *Algebra and Trigonometry: Functions and Application,* less than 10% of the nearly 400 graphs have data points and less 1% of them have data points that do not lie exactly on the function. And in physics classes, teachers often do not discuss how the math in physics classes is different than in math classes—they depend on the math teachers to prepare them. And traditionally taught physics classes who ignore the differences are not working. Traditional (lecture-demonstration) physics instruction produces only small changes in students' naïve beliefs about physics. Hestenes (1992) developed the Force Concept Inventory (FCI), a widely accepted way to probe student understanding of Newtonian mechanics. From pre/post course FCI scores of 14 traditional courses, Hake (1998) found a mean normalized gain of 22%, with a largest single class mean gain of 32%. In contrast, for 41 courses using non-traditional teaching methods, he found a mean gain of 52%, with a largest single mean class gain of 69%.[1] One of the major differences between the traditional and non-traditional physics instruction is the emphasis placed on model building and data analysis.

I believe focusing on data analysis will help students integrate math with physics and will help students understand some of the topics that are not currently being covered. We often do not discuss the significance of the data students collect in labs, and this could be a critical mistake. Tamir (1988) comments that a most frequent error of laboratory teaching in biology, chemistry, and physics is to provide sufficient time for students to conduct the assigned experiment but insufficient opportunity for discussion of the meaning of the experiment and the observations it made possible. Learning in a science laboratory occurs when a group is looking back on the experiments and

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[1] Average normalized gain <g> for a course is defined as the ratio of the actual average gain <G> to the maximum possible average gain, i.e., $\langle g \rangle \equiv \%\langle G \rangle / \%\langle G \rangle_{max} = (\%\langle Sf \rangle - \%\langle Si \rangle)/(100 - \%\langle Si \rangle)$. Where $\langle S_f \rangle$ and $\langle S_i \rangle$ are the final (post) and initial (pre) class averages.

discussing what they mean, why not all results yielded the same values, and how the experiment related to other topics that had gone before. The process of integrating math and science lies at the heart of this process of looking back on the experiments and the data and finding meaning.

The current emphasis in most traditional physics classrooms is on problem solving. Significant challenges arise from this emphasis as Bowden (1992) and others point out: "The capacity to get the correct numerical solution has a low correlation with the capacity to demonstrate qualitative understanding of the concepts in different contexts." A focus on data analysis encourages students to develop models of a situation instead of just finding an isolated numerical solution. Problem solving is important but only as it relates to the scientific model describing the situation the problem addresses. Hestenes (1996) argues that most physics problems are solved by constructing or selecting a model from which the answer to the problem is extracted by model-based reference. And model building and testing is exactly what real physicists do in routine investigations and when they are working to expand the frontiers of our knowledge of physics.

There are many examples of classroom activities that incorporate more of this type of model building and testing; these have been highly successful compared to a lecture-based demonstration. For example, Redish, Saul, and Steinberg (1997) used a microcomputer-based laboratory (MBL) using a sonic ranger in a student tutorial. In this tutorial, students learned how to create and analyze velocity graphs using the real time graphing available with the technology. The students receiving this instruction had error rates of one half to one tenth of similar students who received lecture-based demonstrations. The authors observed that their activities were effective because:

1) Students focus on the physical world.
2) Immediate feedback is available.
3) Collaboration is encouraged.
4) Powerful tools reduce unnecessary drudgery.
5) Students understand the specific and familiar before moving to the more general and abstract.
6) Students are actively engaged in exploring and constructing their own understanding.

These conditions are consistent with modern theories of learning through the process of self-regulation (Lawson 1995). Self-regulation is a reflective process in which the student actively searches for relationships and patterns to resolve contradictions or to bring coherence to a new set of experiences. Learning in science is maximized through this process and facilitated by lesson plans designed with a learning cycle of exploration, term introduction, and concept application (Lawson 1995). During exploration, students learn through their own actions and reaction in a new situation. Term introduction labels the patterns discovered during exploration. In concept application students apply the new terms or thinking patterns to additional examples.

The problems, labs, technology, and software the students will be using in this research help them both to experience these learning cycles in new areas and to apply more mathematical skills in their explorations and applications. For example, the students can use the data analysis software to identify the frequency and speed of sounds they produce using flute-like instruments. Without the software or very expensive equipment, the students would be unable to measure the frequency quantitatively. The software can also give the students immediate feedback on the data they collect by graphing it simultaneously. The software eliminates unnecessary drudgery of repetitious calculations and graphing, but does not fit the data with a mathematical model automatically. This allows the students to test out their own mathematical models and struggle through the essential process of self-regulation.

The process of self-regulation is essential for students to develop more sophisticated beliefs about physics. Without reflecting on their existing beliefs about the world, students rarely change their minds. The students, apparently, are even willing to ignore contradictory evidence if a belief is held strongly enough. Grayson and McDermott (1996) observed this in students working with their computer simulation of a ball rolling on various tracks. The students were given a position vs. time graph of the ball and were asked to construct the tracks and initial conditions necessary to reproduce the graph. They observed that only when students repeatedly failed at achieving the desired graph that they finally set aside their beliefs and tried another approach. When students have a strong belief, telling the students the “right” answer is of little use. Having students analyze data from their own experiments forces them to confront their preconceptions.

However, students have many challenges with data analysis, a critical skill to complete the process of self-regulation in complex physical situations. A large part of data analysis is interpreting the physical meaning of graphs. McDermott, Rosenquist, and van Zee (1987) reported on the difficulties students have connecting graphs to physical concepts and relating graphs to the real world. For example, despite demonstrating a fairly good command of kinematics on problems that did not involve graphs, students had difficulty answering a question about velocity when they were looking at a position vs. time graph. The students were unable to recognize the slope as the velocity. The students were often confused about what part of a graph (slope, height, rise, run, etc.) they needed to look at in order to answer the questions. The students were also unable to translate experimental observations accurately into position, velocity and acceleration graphs and vice versa.

The challenges students face with data analysis extend beyond graphical interpretation problems. Students are unfamiliar with experimental design and have difficulty identifying the appropriate dependent, independent, and control variables. Swatton (1995a) observed that a deep understanding of the relationship between the dependent and independent variables is more fundamental to an integrated understanding of data than the skills necessary to read graphs. Students need to understand what these variables are and how to identify, manipulate and operationalize them. Swatton (1995b) further observed that students' ability to identify the variables in an experiment is highly context dependent: three times the number of students were successful in one context compared to another of similar conceptual difficulty. Students also lack a strong understanding of how to use the concept of variables in related applications. In their computer simulation of ramps, Grayson and McDermott (1996) noticed that half the students tried to modify two variables simultaneously. By doing so, the students obscured the influence of each variable making their task much more difficult.

A student's difficulty with graphical interpretation and variables is likely linked to his/her current stage of intellectual development. Here I do not refer to Piaget's stages of development, but to a new theory of development proposed by Lawson (1995). He starts with the premise that deductive reasoning (i.e. if … and … then) is present virtually at birth, and thus development does not amount to novel changes in this thinking pattern with age but instead involves novel changes to which that thinking pattern can be applied. The critical stages relevant to this research are the empirical-inductive (EI) stage 3 (7 years to Early Adolescence) and the hypothetical-deductive (HD) stage 4 (Early Adolescence and Older). At the EI stage, the child uses deductive reasoning to name, describe, and classify the objects, events, and situations in the child's environment. The thinking patterns associated with this level of thinking include class inclusion, conservation, and serial ordering. The distinct limitation of this reasoning is that it is initiated with only what the child directly perceives in his or her environment. At the HD stage, children become increasingly able to use language to apply the deductive pattern of thinking to "hypothetical" rather than empirical representations. In the HD stage, thinking can be initiated by hypothetical representations by abduction (the process of creating alternative hypotheses). This critical shift allows the child's thinking to be reflective, self-contained, and proactive instead of primarily a response to environmental encounters. The relevant thinking patterns associated with the HD stage include: combinatorial thinking, identification and the control of variables, and proportional thinking. Combinatorial thinking is the process of systematically considering all the possible relations of experimental or theoretical conditions. Identification and the control of variables involves taking into consideration all of the known variables and designing an experiment that controls all but the one being investigated. Recall this is the difficulty Swatton (1995a) observed students had with interpreting graphs in his research. Proportional thinking involves recognizing and interpreting relationships between relationships (e.g. the rate of diffusion of a molecule through a semipermeable membrane is inversely proportional to the square root of its molecular weight). All of these thinking patterns are used in data analysis, especially when students must imagine an explanation for the relationships they discover in their graphs.

A student's beliefs about science may also affect a student's ability to learn effective data analysis strategies. Halloun (1996) has developed a valid and reliable instrument, called the *Views About Sciences Survey* (VASS), that probes students' beliefs about the nature of science and about learning science. He has correlated student beliefs with their achievement in physics classrooms. Students with a more expert profile, a profile similar to a physicist or physics teacher, achieved higher mean grades and significantly higher mean percentages gains in the FCI. Presumably, the students' beliefs will also affect their ability to learn effective data analysis strategies.

Many of the challenges students face when they attempt to analyze data have already been identified. However, it is not clear what all the challenges are when they integrate math and physics through data analysis. Physics and math teachers need to know what the challenges are and how those challenges interact with each other before they can begin to modify their curriculum and pedagogy to help students overcome those challenges. If they do not address these challenges, the students will be left with important gaps in their education. The students will continue to be unable to apply the skills they learn in math to help them understand the physical world, and students will be continue to leave physics classes without much change in their naïve beliefs.

This research seeks to explore those challenges that have already been identified in more depth and to search for more challenges that students may have with the more data-analysis-rich, or quantitative, labs. The students in the study are using a new data analysis program called Fathom that allows students to quickly process and graph their data. Several data-analysis-rich labs have been developed to maximize the benefits of using the software and probe-ware technology to explore traditional and novel topics in new and more powerful ways. This research seeks to answer the following question:

**What challenges do students have when they attempt to integrate math and physics in data-analysis-rich labs?**

# Method

## Setting and Participants

My research was a case study of six data-analysis-rich labs in two different Californian bay area high schools, one in the east bay and one near South San Francisco. The east bay school, Eastwood High (a pseudonym), is very diverse with roughly equal numbers of African American, Asian, Hispanic, and White students. Filipino make up about 10% of the population, about half as much as the other ethnicities. A fifth of the students participate in the free lunch program, and a fifth of the students are English language learners. Roughly 65% go on to college immediately after they graduate and approximately 66% of those students go to community college first. The South San Francisco high school, Oceanbeach High (a pseudonym), has a strong academic reputation. It has a slightly less diverse population which is largely Asian and White. About 10% of the population is Latino and Filipino. 5% of its students participate in the free lunch program and 10% of its students are English language learners. Roughly 85% go on to college and _ of those students go to community college first.

The physics classes at these schools, however, are a small subset of these populations. The physics class at Eastwood High is the only physics class offered and it is an elective. About 15% of graduating seniors take physics. There was an Asian majority with significant numbers of White and Hispanic students. The class consists of roughly 20% sophomores, 40% juniors, and 40% seniors. At Oceanbeach High, there are several different physics classes. There is a conceptual physics class, an honors physics class, and a second year AP physics C class. Approximately 75% of graduating seniors take one of the physics classes at some point. Over half of the honors physics class takes the AP physics B test. The students self-select, largely without guidance, into the honors physics class elective. The students involved in the research were in one of the honors physics classes offered. In this class, there was a significant (60-70%) Asian majority. The class was roughly 50% sophomores and 50% juniors.

Both teachers involved with this research have been paid participants in a project to integrate math and physics through data analysis under a National Science Foundation (NSF) grant for two years. They have been involved in developing and testing the labs and software. The teachers meet with other physics teachers around the bay area every 3 or 4 weeks to discuss and develop lab ideas for the project. The teacher at Eastwood High has been teaching physics for 4 years. This year he moved into a new science classroom equipped with 14 new computers and the latest technology, but he has always had classroom access to computers. He describes his teaching style as traditional with a strong emphasis in using computers and technology to aid learning. He lectures about 70% of the time and does labs 30% of the time. The teacher at Oceanbeach High has been teaching physics for 25 years. His classroom is packed full of student projects, demonstrations, and lab equipment. He has 7 or 8 older computers and has access to the latest technology. He describes his teaching style as a mix of hands-on demonstrations and eliciting and confronting student misconceptions about physics problems. He uses labs as an extension of his hands-on demonstrations, and often asks the students to write up detailed lab reports. He often finds he needs to review and reinforce their math and science skills.

All of the students involved in the study are volunteers. Both sets of students have been given food and T-shirts in exchange for their participation in the project. In addition, the students at Eastwood High are getting some class credit. At Eastwood High, two lab groups, one group of two and one group of three, participated in the project. At Oceanbeach High, one lab group of six students participated. The labs, with one exception, were done by everyone in their respective classes and counted towards their grade.

I tested, observed, interviewed, and elicited the students' opinions in order to get a sense of who they were and how much math and physics they knew. Their basic information as well as their scores on the three tests that I gave them are summarized in Table 1. The three tests I gave are the Force Concept Inventory (FCI), the Scientific Reasoning Test (SRT), and the Views about Science Survey (VASS). A copy of the tests can be found in Appendix A.

The FCI is a multiple choice test that is designed to test the student's conceptual understanding of Newtonian mechanics. It has been used extensively in physics education research, especially in a pre- and post-format to evaluate student learning. The FCI was originally published in The Physics Teacher, March 1992 by Hestenes, Wells, and Swackhamer and revised in August 1995 by Halloun, Hake, and Mosca. In this research I am using it as a way to assess their conceptual understanding of mechanics; however, while the research was being conducted the students were studying concepts not directly related to mechanics. The test could not be used to demonstrate any gain in their conceptual knowledge of physics from the lab activities. The scores do, however, give a way to compare their respective classes and each student's prior success learning Newtonian mechanics conceptually. The students at Eastwood high all scored between

| Student | Ethnicity | Gender | Age / grade | FCI score | SRT score | VASS categorization | Physics Grade | Avg. Prior Math Grade | Current Math Class | Class |
|---|---|---|---|---|---|---|---|---|---|---|
| Amy | Vietnamese | F | 17 / 11th | 32 % | n/a | folk | B+ | B | Pre-Calc | Eastwood |
| Ben | Vietnamese | M | 17 / 11th | 27% | 38% | low transitional | C | B | Pre-Calc | Eastwood |
| Daphne | Chinese | F | 16 / 10th | 30% | 58% | high transitional | A- | n/a | n/a | Eastwood |
| Jo | Chinese | F | 16 / 10th | 23% | 67% | high transitional | A- | A | AP Calc | Eastwood |
| Mica | Asian | F | 18 / 12th | 33% | n/a | low transitional | C | n/a | n/a | Eastwood |
| Jerry | Chinese | M | 16 / 11th | 38% | 63% | folk | D | C | Pre-Calc | Oceanbeach |
| Ken | Chinese | M | 15 / 10th | 85% | 67% | low transitional | A- | A | Pre-Calc | Oceanbeach |
| Laura | Chinese | F | 16 / 11th | 44% | 92% | high transitional | A- | A | Pre-Calc | Oceanbeach |
| Max | Chinese | M | 17 / 11th | 48% | 67% | high transitional | C- | B | Statistics* | Oceanbeach |
| Olivia | Chinese | F | 16 / 11th | 48% | 54% | high transitional | B | A | Algebra 3/4 | Oceanbeach |
| Stephanie | Indian | F | 15 / 10th | 56% | 67% | expert | B+ | A | Algebra 3/4 | Oceanbeach |

| | Representative Quotes from the Students about their Math and Science Backgrounds |
|---|---|
| Amy | n/a |
| Ben | n/a |
| Daphne | “I think math and science requires both logic and common sense. I guess this is why sometimes there can be more than one right answer. In doing a math problem or science experiment, it is important to follow the directions and go one step at a time.” |
| Jo | “…my parents have always told me that I am quick to learn and catch onto whatever my teachers taught. As a results, I am now steps ahead in my math and science paths of high school.” |
| Mica | n/a |
| Jerry | “Taiwanese math teachers are very strict, so I learned a lot back in Taiwan.” *(he left after 6th grade for the US)* |
| Ken | “I think that [math and science] is very important and interesting at times, especially science.” |
| Laura | “For mathematics, I have always enjoyed doing math problems, and I guess I have been pretty much at the top of the class every year.” |
| Max | “[My parents think] that [Math] is very important to learn and to get good grades on (I am currently not)” |
| Olivia | “Before I had him as my math teacher, I did not really have that much interest in math. I did not dread the class, but I was not excited about it. This math teacher made math fun and made me think about math as a career.” |
| Stephanie | “I think I share my mother’s opinion in the thought that if you are good in math, as long as you try, you will also be good in most, if not all, your other subjects.” |

* Max was dropped from Pre-Calculus because he was in danger of failing.

Table 1.Summary of test results and quotes from students involved in the research.

20-30% on the FCI, which is not unusual for traditionally taught classes. The incorrect choices on the test are very attractive because they represent the common misconceptions most people have about mechanics. These results indicate that the Eastwood students in the study do not have the same academic background, overall academic orientation, and/or have had more difficulty learning physics conceptually. At Oceanbeach High, the average results (53%) are higher, but the there is more variability from 38% to 85%. The Oceanbeach students may have had a stronger academic background and/or a stronger overall academic orientation. They may also have been benefiting from the Oceanbeach High's physics teacher's greater experience and focus on eliciting and confronting their misconceptions.

The SRT is designed to test aspects of scientific and mathematical reasoning that are necessary to analyze a situation, make a prediction, or solve a problem. This test is one way to measure a student's intellectual development. It examines a student's ability to think hypothetical-deductively (see page 6 for more details). From the results shown in Table 1, the students had a wide range of scores from 38% to 92% with an average of 64%. On a few questions the majority of the students did not get the correct answers. The last four questions described the results of an experiment and proposed a hypothesis as an explanation of the results. Then the questions asked what experiment they would do to test the hypothesis and whether the results proved or disproved it. The students scored an average of 33% on these questions (and the students did not score more than 50% on any one of them). The students also had difficulty with questions 11-14, scoring an average of 39%, where the students were asked to come up with a hypothesis that was consistent with the results of an experiment. Based on these results, most students are having trouble applying two HD thinking patterns (combinatorial thinking, and identification and control of variables), at least in these specific contexts. Most of the students are in various stages of transition between EI and HD thinking patterns.

The VASS is designed to identify the attitudes that affect how people learn and understand physics. The designers of the survey compared the students' answers with physics teachers' and physicists' answers. Based on how close students' answers were to the physics experts' answers, they categorized the students into four categories: folk, low transitional, high transitional, and expert. As you can see from Table 1, there was a significant diversity of attitudes about physics among the students. Only one student shared the vast majority of the beliefs of physics teachers and physicists, five students shared most of the “expert’s” beliefs, three students shared many of them, and two shared few. On most of the beliefs tested, the students gave a diversity of answers with a few exceptions. Six of eleven students believe that in physics, mathematical formulas provide ways to get numerical answers to problems more than they express meaningful relationships among variables. Four others believe mathematical formulas do both equally, and only one student believes the opposite as the experts do. The majority of students also believe that theories in physics will be maintained as they are and could not be eventually replaced by other ideas. This is contrary to physicists who leave room for the possibility that the existing ideas can be eventually replaced. The creator of the VASS, Halloun (1996), has correlated student beliefs with their achievement in physics classrooms. He found students with a more expert profile achieved higher mean grades and significantly higher mean percentages gains in the FCI. The small sample of students involved in this research seems to confirm this pattern.

In addition to these tests, I asked the students to write answers to several open ended questions about their past experiences and influences in math and science. In Table 1, I have taken out a representative quote from each student. Overall the students have positive attitudes towards math and have received good grades in mathematics with a few exceptions.

## Data Collection Strategies

In order to examine the challenges the students had integrating math and science with data analysis, six data-analysis-rich labs were used covering six different topics in physics: Ohm’s Law, Internal Resistance, Snell’s Law, Conservation of Energy, Projectile Motion, and Constant Acceleration. Data-analysis-rich labs focus the students’ attention on analyzing data, creating mathematical models, and either comparing those models to existing theory or creating explanations themselves for the relationships they find. These labs were developed under a two-year grant from the National Science Foundation (award number DMI-0216656). Copies of all the lab activities can be found in Appendix B.

I collected a number of different types of information from the labs. First, I videotaped the students doing the labs. I later examined each of the videotapes, paying close attention to the areas that seemed to cause the students the most difficulty. I often transcribed those areas for closer analysis. Second, I examined their lab reports and their answers to the questions the lab activities posed, looking for common mistakes and misconceptions. Third, after examining the videotape and their written work, I interviewed the students as a group asking them for their opinions about how the labs went and following up on any difficulties they had. Finally, I had a brief interview with each student asking them for their general impressions of the labs we did together.

## The Conceptual Framework

Based on my observations and the literature, I have developed a conceptual framework to help understand all the factors involved when students attempt to integrate math and physics in data-analysis-rich labs. Figure 1 presents a diagram of the integration model. The three circles represent the students' data analysis skills, their equation fluency, and their perspective. These terms will be defined in the next section. The overlap indicates success at times depends on data analysis skills and equation fluency, equation fluency and perspective, and data analysis skills and perspective; at times all three elements are required. The context of the lab and their classroom influences how successfully they apply and develop their data analysis skills, equation fluency, and perspective. The success students have integrating math and physics in data-analysis rich labs is the product of the interplay the students' equation fluency, data analysis skills, and their perspective in the particular context of their classroom and the lab.

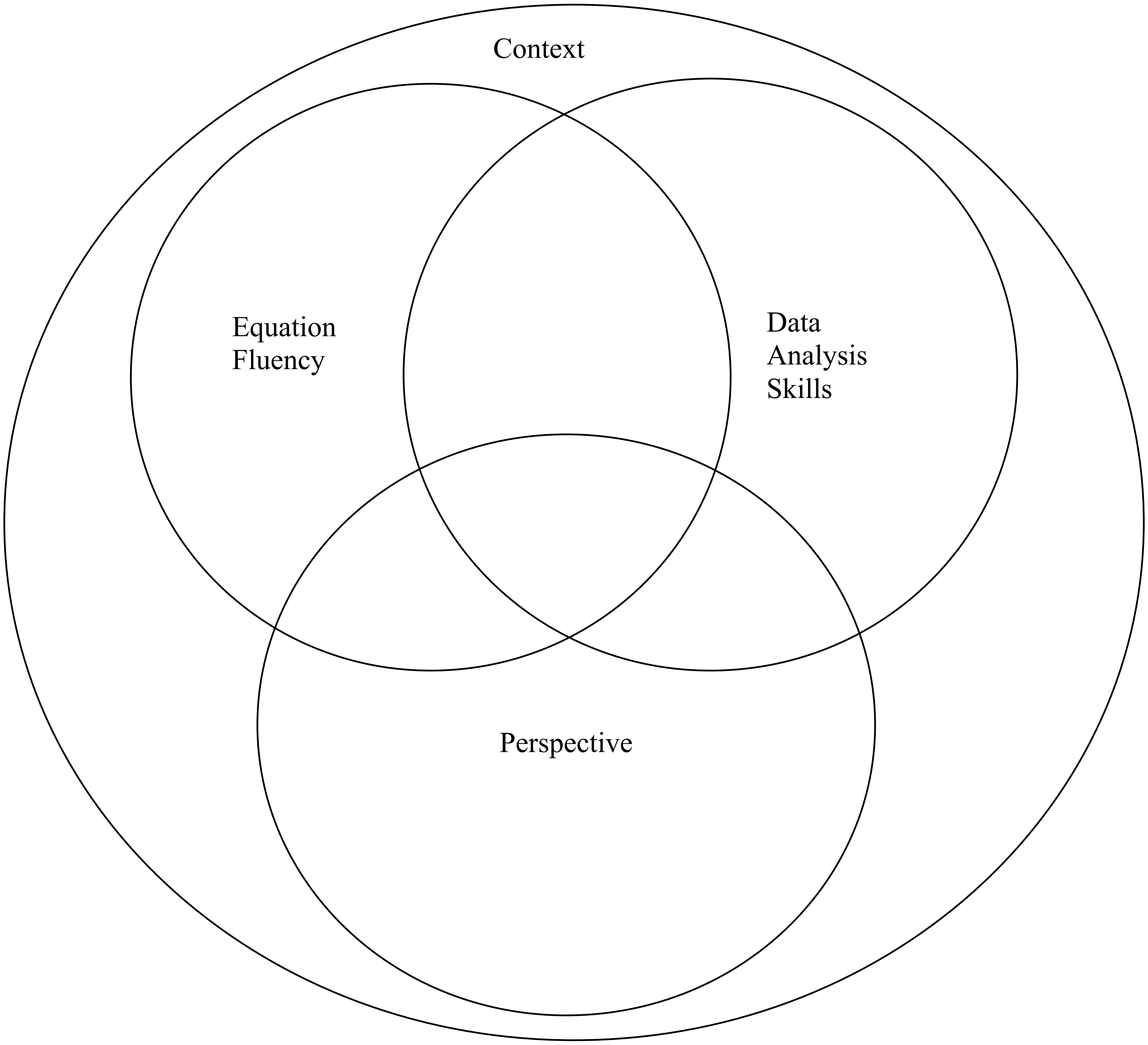


**Figure 1.** Context, Equation Fluency, Perspective, and Data Analysis Skills are all involved when students attempt to integrate math and physics in data-analysis rich labs.

# Definition of Categories

## Equation Fluency (Adkins 2004)

Equation fluency represents how well a student can manipulate and read an equation for understanding and significance. Many students do not seem to have this fluency coming into physics class. With good reason, the math in physics is often significantly different than the math the students study in math classes. First, the mathematical language in physics class is often different. In math classes, the variables are almost always labeled “x” and “y” whereas in physics class the variables can be any English or Greek letter. Constants in math class are rarely represented in equations as letters, and when they are they are usually only “a”, “b”, “c”, and “m” (for the slope of a line). In physics class different letters frequently represent constants, such as g = 9.8 m/s$^2$. And those constants appear in equations right next to numbers, such as the equation to find the distance a object fell in a certain amount of time, given below:

$$d = \frac{1}{2} g t^2 + v_i t$$

In this equation the constant in front of $t^2$ is a combination of a fraction _ and g, a letter representing 9.8 m/s$^2$.

The differences go beyond just using different letters in unique ways. In physics the letters represent either a description of the structure of a physical system or its properties. The same letter might mean something different depending on the model being used. People use the word “model” is many different ways, so I want to clarify how I am using it. I am using Hestenes’s (1996) definition of a model in physics as “a representation of structure in a physical system and/or its properties. The system may consist of one or more material objects or massless entities such as light.” He went on to describe the four different types of structures that make up a model (systemic, geometric, temporal, and interaction). He argues that “…complete understanding of a model requires *coordination of multiple representations* [of the different structures making up the model].” (emphasis in the original). One equation that defines the temporal structure of a mass experiencing a force is F = ma. Even if students understand the formula mathematically, they may not understand that the force, F, represents the net force on one object in the physical system they are studying. So they may correctly apply the rules of mathematics, but still fail to solve the problem accurately. Therefore, understanding the meaning and significance of equations in physics requires that students understand the underlying model. In contrast, the equations used in traditional math class often have no physical meaning at all.

Equations in physics also differ from equations in math class because they often have more than two variables. To get all the meaning expressed in an equation such as this, a student must know how to separate out the relationship of any two variables in it.

To do this they must imagine the other variables to be constant and sometimes must rearrange and simplify the equation. For example, to understand all of the meaning of Newton's Second Law ($F = ma$), a student must be able to predict the relationship between: F and a; F and m; and m and a. (We will ignore for now the complexities behind summing all the forces and the vector nature of F and a.) In order to predict and understand the relationship between force, F, and mass, m, a student must hold acceleration, a, constant. In this case F and m are directly related. This means more force is required to accelerate a larger mass the same amount as a smaller mass. m and a, however, are inversely related. If force is held constant, less mass will accelerate more or more mass will accelerate less. For full understanding the student also needs to know how the quantitative relationships look like on a graph, especially if they are going to link their understanding to the results of an experiment.

Additionally, the variables and constants in physics represent physical properties or descriptors, so they are represented by both a number and a unit. So students must be familiar with the common units and their abbreviations as well as how to convert units. An understanding of units can also reinforce a student's equation fluency because the units must be the same on both sides of an equation. Therefore, a student can check for unit consistency in their equations to confirm their equations.

Despite these differences, physics can frequently appear very similar to math. The problems students are asked to solve are frequently very similar to the problems the teacher solves as examples. Therefore, the students can have a good deal of success in many physics classrooms just learning the right procedure to apply to each different type of problem *without understanding the physical meaning or significance*. Essentially, the students learn to treat physics in a similar way as procedure based math class: learn the right procedure then apply it to find the right numerical answer. These students will run into trouble when the situation is changed slightly, which in turn changes the underlying model, and therefore the equations describing it and/or their meaning. Students with equation fluency will read an equation for meaning and significance, not just use it to solve for numerical answers. Equation fluency often depends on a student's perspective and their data analysis skills which are both defined later.

## Data Analysis Skills

The data analysis skills category includes all the skills students have that allow them to collect, to process, and to interpret data accurately and purposefully. In order to collect data accurately and purposefully, the students need to be familiar with the purpose of the lab and how the data will be analyzed and used. For example, if the students want to verify that force, F, and acceleration, a, are directly related they will likely not measure F and a directly. They might use time and distance and later calculate the acceleration, a, from the relationship below:

$$d = \frac{1}{2}at^2$$

where d represents the distance the mass moved in an amount of time, t. Other labs require students to measure even a distance indirectly. For example, in the Pendulum and Energy Conservation lab the students did for the research, they needed to measure the distance from the top of the swing to the bottom. It is hard to measure the distance from the top of the pendulum swing to the bottom directly. So the students measured to the top of the swing from the table and were expected to measure to the bottom of the swing from the table and subtract the two to get the distance represented by "height" in

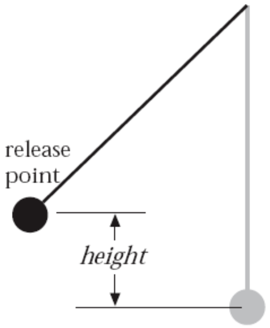


Figure 2. Diagram of height students needed to measure indirectly in the Pendulum and Energy Conservation lab.

In each of the above cases, it is important for students to record their original measurements, so they can correct any mistakes in their derived measurements, due to a calculation error.

Students must also be aware of how to recognize and minimize any error in their measurements. For example, students will often do multiple trials for the same initial conditions in order to get a sense of the variability and error in their measurements. Most commonly, time measurements are taken multiple times in order to get either a more accurate measurement by averaging them or a sense of the variability. If students can estimate the likely error their measurements will have, they will also be more likely to recognize when they make a mistake recording or measuring their data. In order to estimate the measurement error, students need to understand how to use various measurement tools and in some cases how the measurement tools work.

During the process of data analysis students often also need to understand the impact measurement errors or variability can have on their ultimate results. In some labs, students must determine a parameter like g and compare their data to the expected values or to other students' results. Students must understand the variability of their data in order to find a reasonable range of values for a parameter. In other cases, measurement error and variability may completely obscure the relationship between the two variables studied. Judging when data does or does not fit a model can be very difficult without a sense of the variability in the data. Students need strategies to estimate the error in their measurements and measure the variability in their data. One simple way to gauge the variability in their data is to take multiple measurements at the same values. In this way students can get a range for each of their data points and more accurately judge what functions might reasonably fit the data they have.

Related to measurement error is the concept of units. Students need to have a good grasp of the concept that any measurement needs both a number and a unit in order to be meaningful. In the U.S. the students must know the common units in both the English system and the SI (International Standard) system, which uses metric units. In addition to knowing the names and abbreviations, the students need to have a feeling of the scale of each common unit in order to catch their own mistakes. In one lab I observed a student who recorded the distance a projectile in class went as 363 m instead of 363 cm. If the student had a feeling of the scale of a meter he would have caught his error immediately instead of later getting wildly inaccurate results in his analysis. Ideally, students must also know how to convert units and know the common units of force, distance, time, acceleration, etc. in both the SI system and the English system.

Data analysis includes the construction and interpretation of graphs. In this study, students used computers and a program called Fathom to help them construct their graphs. Therefore, the students needed to know how to record, process, analyze, and graph data using Fathom. This included getting data from probes, making a case table, making computed attributes in a case table, using sliders as variable parameters and constants, making graphs, graphing functions on graphs, and constructing residual plots. Students also needed to understand how Fathom used and displayed units and how Fathom could use whole words instead of just letters to represent variables and parameters. By learning how to use Fathom, the students were able to quickly process large amounts of data, make graphs in seconds, and match their data with mathematical functions in minutes, giving them more time to interpret the meaning and significance of the graphs, parameters, and functions. The students had to understand how scaling on the axes could affect the appearance of their data. Fathom will automatically scale the axes, which can lead to some misunderstandings if students are not aware of this feature. A small amount of data also can obscure the relationship between the variables because many relationships such as a square root relationship can look like a linear relationship, or a parabolic relationship can look like an exponential relationship. This is especially true if the students do not appreciate the variability in their data or do not have equation fluency.

## Perspective

The perspective category represents how the student perceives the process of integrating math and science through data analysis. Perspective includes the student's beliefs about math and physics as well as their intellectual development. Students' attitudes and beliefs were compared to physics "experts" using the Views about Science Survey (VASS) survey. Depending on a student's attitude toward physics and math, he/she may view the purpose of math, physics, equations and labs differently. There was a significant diversity of attitudes about physics among the students (see Table 1 on page 9 and pages 10-11 for more details on the students' answers).

Their intellectual development also plays a significant role in how the students approach the labs (see page 5 for more details). Their intellectual development was assessed with Lawson's (1995) Classroom Scientific Reasoning Test (SRT). (see Table 1 on page 9 and page 10 for more details on how the students in the study performed). In order for students to analyze the data from the labs, the students need to understand the model of the physical system they are testing. The models are complex representations that involve the relationship of many variables. This clearly requires combinatorial thinking, proportional reasoning, and identification and control of variables—three HD thinking patterns Lawson identified. For example, the students' scores on the FCI indicate that most of the students do not accept or do not understand the Newtonian mechanics. In order for the students to replace their own theories with others, they need to be able understand how the model represents the physical system. In other words, they need to identify their own mechanics misconceptions and use deductive reasoning to predict some consequences of those misconceptions and/or of concepts in Newtonian mechanics. Then, they need to do or see experiments testing those predictions, and use their equation fluency and data analysis skills to interpret the results successfully.

## Overlap of Equation Fluency and Data Analysis

Equation fluency and data analysis skills overlap when students are asked to find the relationship between the independent and dependent variables in their experiments. If the students already know the equation or model they are testing, equation fluency allows the students to predict the relationship between the two variables. This facilitates the process of matching a function to their data because they can start with at least the right form (linear, quadratic, etc.). If the model or equation is not known prior to the experiment, the students with equation fluency are at least familiar with the common forms relationships can take, and so they can more easily identify and match a relationship to the data. Even if a student recognizes a parabolic shape in the data, if they do not have equation fluency it is hard to express the relationship mathematically. Conversely, if the students do not have the data analysis skills to recognize the problems of variability, scale, and limited range with data, they may not be able to recognize the relationship or erroneously conclude the model is incorrect.

Another area where equation fluency and data analysis overlap is with units. If students are looking for a parameter like g, they may get a result of 1000 instead of 9.8 .

The number they found is likely in cm / $s^2$ instead of the standard units of m / $s^2$. The number is reasonable, but the students may not recognize it as correct. In other cases inconsistent units will cause them to get inaccurate results.

## Overlap of Equation Fluency and Perspective

Equation fluency and perspective support each other in a number of areas. A student's willingness to find the significance and meaning of an equation has to do with their attitudes about physics and math. For example, do the students believe that they will use the content knowledge and intellectual skills they develop in physics in their future career or everyday life? If they do, they will likely spend more time and energy looking for the meaning in an equation rather than just as a means to an end (like the numerical answer to a homework problem).

All three hypothetical deductive thinking patterns (combinatorial thinking, identification and the control of variables, and proportional thinking) are critical for equation fluency. In order to understanding the meaning of equations, a student must be able to identify and control variables, systematically identify the relationships between those variables, and recognize the relationships between relationships. Proportional reasoning is a particularly critical skill for equation fluency. Lawson (1995) defines proportional thinking as the ability to recognize and interpret relationships between relationships in situations described by observable or theoretical variables. For example, if for every 12 banded frogs there are 72 total frogs, for every 55 banded frogs, there must be 330 total frogs. One of the important markers of intellectual development is the ability of the student to use proportional reasoning.

## Overlap of Data Analysis and Perspective

Acceptance of a scientific perspective and all three HD thinking patterns Lawson (1995) identified are critical for data analysis. A scientific perspective makes data analysis meaningful. If a student is happy to accept explanations of how the world works based on what an authority figure says, they may find data analysis tedious and difficult. Or if a student's perspective is focused on getting a good grade and data analysis is not represented on their assessments, a student may not put much effort into the data analysis work. A scientific perspective relies on experiments and evidence to support or disprove explanations of the natural world. Students who adopt this attitude will be more engaged by a lab activity and more motivated to learn the finer points of data analysis. Even if students have the right attitude, the still might struggle with data analysis because they do not have the necessary HD thinking patterns. In any quantitative data analysis task, students must identify and control for variables. Combinatorial thinking, the process of systematically analyzing the relationship between all the variables involved, is often necessary, especially in unfamiliar contexts. And proportional reasoning is required to anticipate and identify the relationships data analysis often reveals.

## Overlap of all three

Successfully interpreting a graph often requires the alignment of all three categories. The student must understand the experiment and the hypothesis they are testing, which often requires hypothetical-deductive reasoning. In regards to data analysis, the student must know how to collect, process, and interpret the data. In addition, the student needs to understand what the axes represent and the scale of those axes. The axes represent physical properties and descriptors whose meaning again depends on the model of the physical system they are examining. For example, if the students have found a line that fits the data of force (F) vs. acceleration (a), the slope represents the mass (m) of the object being accelerated. A student with equation fluency will readily understand this, but any misunderstanding of the model that defines F, a, and m or the data may lead to misinterpretations. For example, F in the model represents net Force, and if the students did not measure net Force their data may not fit on a line, and even if the data fits on the line, the slope may not be the mass they think. The student would likely also have had to calculate acceleration indirectly, requiring data analysis skills. The students may encounter problems setting up the experiment and might be required to troubleshoot their data collection process. The error in the data may also be significant enough that a linear relationship is not obvious. So the students may need to assess if the error is still reasonable enough to accept a linear relationship. Finally, a student without a scientific perspective may not fully understand the purpose of the experiment. A student without a firm grasp of proportional reasoning may not fully understand the implications of the experimental finding.

Designing and implementing their own quantitative experiments also usually requires skills from all three categories unless the context is very familiar. Many students are capable of performing and designing qualitative experiments in a number of different contexts because they can adopt a scientific perspective. However, in order to design and interpret a quantitative experiment, they must have some equation fluency and data analysis skills as well as the ability to adopt scientific perspective. In any quantitative experiment, the student usually must interpret a graph to test their quantitative hypothesis, and so all the skills that apply to interpreting a graph must apply to designing and implementing their own quantitative experiments.

At the highest level, students can go beyond the models presented in the physics textbook and explore reasons for small secondary effects they can discover in their data with more advanced data analysis skills and the ability to think of new hypotheses. For example, in an experiment to test the conversation of energy, students might examine how the potential energy of the pendulum bob is converted into kinetic energy. By manipulating the equations of the model, they might predict the relationship of velocity to the height the bob was released from is $v = \sqrt{2gh}$ . When they collect their data they could confirm that this relationship is true; however, students might go further and ask why their data predicts a lower value of g than the accepted value (maybe there was some energy loss due to friction). They might also notice some systematic error in their data because they did not measure the height using the bob's center of mass. Using residual plots—graphs of the differences between their functions and their data—students can often explore these subtler effects.

## Context

The context influences how successfully students apply and develop their data analysis skills, equation fluency, and perspective. The context includes, but is not limited to, the contents and sequence of the lab in the curriculum, the teacher, the teacher's pedagogical style, the school's expectations, teacher's expectations, the student's expectations, their parent's expectations, the class environment, the group dynamics of their specific lab group, accessibility to outside assistance, and the individual student's past experiences with math, science, and data analysis. Due to the limited nature of this study, I did not have time to explore all of these possible factors. I will only speculate on the effects of the sequence and contents of the lab, the teacher's self reported pedagogical style, and the students' past experiences with math, science and data analysis.

The sequence and contents of the lab influence how successful students will be. This includes how much and how the material was introduced to the students prior to the lab. Lawson (1995) suggests that to optimize student learning the curriculum should be introduced in a series of learning cycles. Learning cycles consist of three phases: exploration, term introduction, and concept application. There are three type of learning cycles: descriptive, empirical-abductive, and hypothetical-deductive learning cycles. Quantitative labs, like those in this study, are usually part of hypothetical-deductive learning cycles. During the exploration phase of a hypothetical-deductive learning cycle, the students explore a phenomenon and, as a class, discuss hypotheses, deduce implications, and design experiments. Then the students conduct the experiments. During term introduction phase, the data is compared and analyzed, terms are introduced, and conclusions are drawn. During the concept application phase additional phenomena that use the same concepts are discussed or explored. The specific contents of the lab also influence how successful students are. Their ability to think hypothetical-deductively is highly contextually based. The ability to identify variables and come up with explanations for relationships between them is necessary for HD thinking patterns. It seems reasonable to assume that a student's ability to do this is dependent on how familiar the student is with the material. This is suggested by Swatton's (1995b) observations that students' ability to identify the independent, dependent, and control variables in an experiment is highly context dependent: three times the number of students were successful in one context compared to another of similar conceptual difficulty.

A teacher's pedagogical style is also very important in preparing students to learn the conceptual models in physics. Traditional lecture and demonstration style teaching has been shown by Hake (1998) to have a significantly lower impact on student's conceptual understanding compared to a modeling-based teaching style. Are the students being explicitly taught to process and present the material to each other and the teacher using models? Or is the instruction focused on getting the students to solve problems using the formulas they learn through lectures and demonstrations?

A student's past experience with math and science also determines, in part, their equation fluency, their perspective, and their data analysis skills. Are they familiar with units? Are they familiar with the common relationships between two variables and their graphs? Have they used Fathom before? How often did they work with real data in math class? How often did they consider the error and variability in their data in prior science classes? Do they see math and physics as important in their future careers and/or lives?

## Case Studies

I will now turn to the case studies of each lab to illustrate and test the relevance of this conceptual framework. The aim of this research was to identify challenges students have with data-analysis rich labs. Using the broad categories in the math and science integration theory described above, I will examine each lab experience for evidence of difficulties the students had with each one. Three labs (Ohm's Law, Farley Effect, and Snell's Law) were done at Oceanbeach High. Three more labs (Pendulum and Energy Conservation, Mars Needs Rubber, Constant Acceleration on a Ramp) were done at Eastwood High.

### Ohm's Law Lab (Oceanbeach High)

The purpose of the lab was for the students to find the relationship between voltage and current in a simple circuit with a constant resistance. As extensions the students explored the effect of setting up two resistors in series and in parallel. Before the lab started the students predicted the relationship between voltage and current. The students were introduced to the concepts of voltage and current but not resistance prior to the lab. The students set up the lab and took data while being videotaped and recorded. The students took data on paper as well as on the computer. During the lab, the students generated graphs of the data using Fathom. They graphed current vs. voltage and voltage vs. current. The students also fit their data using a moveable line. The students collected data on series and parallel circuits on their own time at school the next day, and finished their lab reports later at home. A full copy of the lab pages the students used can be found in the Appendix B.

All of the students prior to the lab predicted that the current would go up with voltage, but the increase would slow as it approached some limit. During the lab, the students had a lot of difficulty with the equipment. Some of the equipment was not working at first, and it was easy to have a connection come loose. For all the students this was the first time they had put together circuits. It was also their first time using multimeters to measure current and voltage. To make matters worse, there were two different types of multimeters with different markings. The circuit they built was a simple circuit, but with real wires and unfamiliar equipment the students took 15 minutes to build the circuit and get somewhat familiar with the equipment. Once the students got the circuit working well, they took data for several different resistors and graphed the results on Fathom.

The students had several challenges with this lab both during the lab and after. The first challenge came up when they were using the multimeter to measure a small current. The multimeter has several different settings to measure current, and each setting allows greater precision. The maximum current each setting can measure is displayed on the multimeter next to each setting. The finest setting displays the current in _A while the other two settings displays the current in mA. All but one of the students, Max, did not want to use the different settings because they did not want to mess up their data. They did not seem to understand that the different settings showed the same current, just in greater precision and with the finest setting in different units. The second challenge involved a question about what the slope represents when you flip the axes of the graph of Current vs. Voltage to Voltage vs. Current. The following is how the students' conversation went after they just made the first inverse graph:

…laughter
Jerry: "It's the same thing"
Olivia: "Did you switch it? did you switch it?"
Jerry: "Yeah, current is on the bottom, voltage is on top"
Olivia: "Ohhhh"
Olivia: "It should be the same, shouldn't it?"

Laura: "Did it tell you to do that?"
Olivia: "Yeah, reverse."
Ken: "Oh you know what, it is like a inverse. It is the inverse"
Olivia: "So the same thing, right?"
Stephanie: "No, it's not the same thing, no, the inverse of the function is not the same thing."
Laura: "Yeah it is sometimes"
Olivia: "Try another graph, she did not believe that one"
Stephanie: "Wait, 'm' is going to be the same exact for both graphs?"
Ken: "No, it's inverses."
Olivia: "But the graph is the same"
Ken: "The graphs are similar but they are differences. Can we get a line on the graph and have it show the equations?"
….(interruption by the teacher with more instructions to disregard negative voltages)…
Stephanie: "It can't be the same it would be rise/run and run/rise."

The students later confirmed that the slopes are different on the two inverted graphs. Fathom autoscales the axes, so when they flip the axes the graphs look the same. Ken, Stephanie, and Laura were trying to explain this to the others when the class ended.

Later when the students were analyzing their results, they found the different slopes for each of the graphs for each of the different resistors. Four of the six students were careful to report that the equations they found predicted current in [mA] based on a given voltage. Two students, Ken and Jerry, did not. Ken used an example in his report that predicted a current of 26 A with a voltage of 26 V. He may have realized his error if he had a sense of how large a current 26 A really is.

On the next day the students came in on their own time to test parallel and series circuit in a similar way. The students got poor results due to equipment problems, so they used another group's results in their analysis. Laura described why she discarded the data: "After examining our own data, though, we found that the graphs were extremely odd. It did not make sense to have more than one point on the same line (vertical line test)."

The students analyzed the data on series and parallel circuits, but only after they saw a video in class describing why they are different. The video described why in series circuits the resistance increases and parallel circuits the resistance decreases. The video used the relationship:

$$R = \rho \frac{L}{A}$$

where:
R is resistance
_ is resistivity
L is length
A is area

The video explained that a parallel circuit has more area, so the resistance is lower, and a series circuit is longer so the resistance is higher. The majority of the students took this explanation and erroneously applied it to their results with the series and parallel circuit. This formula does not directly apply to the series and parallel circuits they used. It refers to wires of various materials, lengths and areas, not to the resistors they used in the lab. Olivia described: “Since length and area for each resistor is different, the quotient that represents each resistor varies.” The quotient she refers to is the slope they found. So she suggests the differences in slope came from different lengths and areas of the resistors, but all of the resistors were the same size in the lab. Stephanie used the equation to describe why the series resistor had more resistance: “In a series circuit, L is increased since there are 2 resistors instead of one. This means that R should be increased since R and L are proportional according to the above equation.” She noticed that this was inconsistent with the previous data, however: “But according to our data, the resistancy of one of the single resistor circuit seems to be more than the resistancy of the double resistor circuit most probably because of human error during the experiment.” Here it seems the student did not understand the underlying model and misinterpreted her results as incorrect, so that she could match them to her misinformed understanding.

### Farley Effect Lab (Oceanbeach High)

The purpose of the Farley Effect Lab was to find the internal resistance of a battery. The students were given the pre-lab questions the class before to prepare them to analyze the circuit they would be using in the lab. I learned later that the students did not do the pre-lab before the lab. However in lecture the students saw an example problem, where the teacher solved for the internal resistance of a battery. Due to equipment problems, the students ended up only analyzing the data another group collected. The students again graphed voltage against the current but for a slightly different circuit using a battery instead of a variable voltage source shown below:

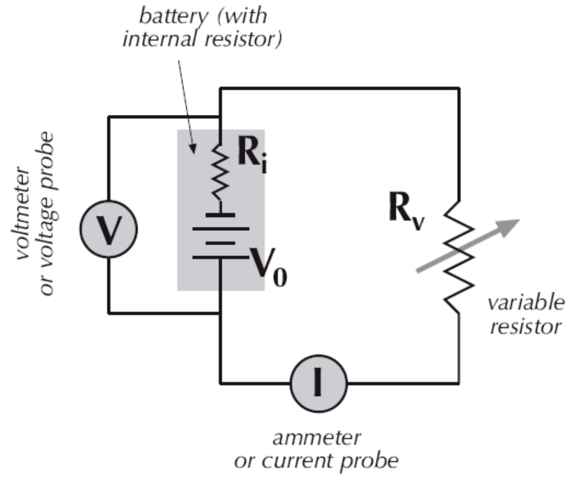


Figure 3. A circuit diagram representing the circuit the students used to determine the internal resistance of the battery in the Farley Effect Lab.

The students graphed the voltage measured by the voltage probe against the current measured by the ammeter. The resistance in this circuit was varied directly instead of the voltage as was the case in the Ohm's Law lab. In this circuit changing the resistance will increase or decrease the current and that will change the voltage measured by the voltage probe. This lab uses Ohm's Law, but in a different way than the first lab the students did. The pre-lab questions were designed to guide the students through the internal resistance hypothesis and the implication that the measured voltage, V, should be related to the internal resistance, $R_i$, and the current, I, according to this equation:

$$V = V_o - I * R_i$$

A full copy of the lab is in Appendix B.

Only Max, Ken, and Stephanie were there the day the lab was videotaped. From the very beginning of this lab, the students were confused. The students quickly made a graph of V vs. I. Ken looked at the graph and noticed the negative slope and commented:"so what it means is that when you go down in voltage you go up in amps, current…that's weird." This was not what he expected, so he looked back to the circuit and asked: what was the same for all the data. I replied that the same battery was used in all the data, so the relationship between voltage and current must somehow tell them about the internal resistance of the battery. I also mentioned that there were a lot of hints in the lab if they needed help. Stephanie read that the intercept represents the true voltage of the battery. Ken commented : “that means it won’t have any internal resistance, right?” They looked at the graph again and Ken found the y-intercept and said : “okay, so this is the true voltage, but then the current is zero, I do not get it”. Later he said: “Can you explain this to me, if the current is decreasing, why is there more voltage?” Ken continued to apply the model he developed from the Ohm’s Law lab: voltage is directly proportional to the current. He did not seem to understand how this circuit was different than the circuits they explored in the first lab. Stephanie later showed she shared his confusion with the comment that: “as E [voltage] increases I [current] should increase shouldn’t it?” This confusion lasted the entire lab, despite my attempts to help them interpret the situation.

During their attempts to understand the lab, they demonstrated other difficulties. At one point, while looking at the graph she commented: “V and I are directly proportional, no inversely proportional.” It seems she misinterpreted a negative slope as an inverse relationship. Ken also had trouble reading the equation of the line displayed on the graph because Fathom displayed the units, represented by V and A, in the equation along with words to represent the voltage and current. He could not interpret the function because he was not familiar with seeing units in an equation and/or he was not familiar with the units for voltage and current. Another contributing factor was a notation difference for voltage between the teacher and the lab write-up. The lab represented voltage differences by V and the teacher represented them by E. After an hour of struggling with the data, Ken was able to find and explain how to find $R_i$ using one data point and $V_o$ after I described it to him. However, he still said: “but the thing I do not really get is why dividing V by I isn’t $R_i$ plus $R_v$.” He could repeat a problem solving process I described to him, but he never understood that the voltage they were measuring was the voltage drop across the variable resistor, $R_v$. At this point, Max showed Ken how the numbers worked out and Ken dismissed his contribution. Max threw up his hands and appeared frustrated with Ken because he was satisfied that all the numbers worked out and could not understand why Ken was not also satisfied by this.

Max and Ken continued to struggle with the concepts in the lab two weeks later. They were interviewed about the lab and Max admitted he did not understand the lab and could not express V in terms of $V_o$, I, and $R_i$. His answer was: "V=IR". Ken's answer also indicated some lingering confusion: "$V_o$ is the true voltage w/o the internal resistance / $V = V_o + IR_i$ (This is slightly incorrect, the relationship should be $V = V_o - IR_i$). In response to the question: what would the graph of V vs. I look like if there was no internal resistance? Ken answered, "Same thing? Not sure." It shouldn't be the same thing. I think Ken was still struggling with a limited model of electrical circuits.

Laura, Olivia, and Jerry did the lab later on their own. They answered the pre-lab questions and the lab questions, but they limited their answers to mostly just numbers. Their answers on the pre-lab questions were wrong, suggesting they also had a hard time analyzing the circuit. Their answers to the lab questions were correct, but they did not explain their answers and never appeared to use the relationship: $V = V_o - I * R_i$ to model the data they used.

### Snell's Law Lab (Oceanbeach High)

The purpose of this lab was to find the index of refraction of an acrylic (a type of plastic) block. Light bends as it passes from one medium to another depending on the medium's index of refraction. This lab was similar to the Farley Effect lab because the students were only asked to analyze the data and find a parameter rather than collecting the data themselves and discovering a relationship. In this case, the students saw a video demonstrating how the data was collected and they were given the relationship between the incident angle and the refracted angle. The video quality was poor due to excessive light in the room, and many students may not have been able to see it clearly. Prior to the introduction to the lab they had not been introduced to Snell's law (given below), the law describing how light bends.

$$n_i \sin\theta_i = n_r \sin\theta_r$$

where:
$n_i$ is the index of refraction of acrylic in this lab
$\__i$ is the angle of incidence of the light
$n_r$ is the index of refraction of air in this lab

_$_r$ is the angle of refraction of the light

The students had about 20 minutes to finish the lab, and the lab immediately followed a physics test. A full copy of the lab is in Appendix B.

The students had a few difficulties with this lab. The students had difficult calculating the sine of the angles using Fathom. The majority of the students also had challenges interpreting the meaning of the slope of the graph of the sine of the incident angle vs. the sine of the refracted angle. The students also had some trouble interpreting the slope of the inverted graph.

The students were given the incident angle and refracted angle for several different light beams. The students had to convert the angles into radians and calculate the sine of those angles using Fathom. The students were unfamiliar with how to enter units and functions in Fathom and got some help entering the units and calculated attributes of iSin, sine of the incident angle, and rSin, sine of the refracted angle.

Once the students had iSin and rSin calculated, they graphed rSin vs. iSin. The students encountered their first data analysis challenge when they attempted to answer: what does the slope of your line represent? The students' discussion follows:

Stephanie: "It represents the ratio of $n_i$ to $n_r$"
Olivia: "Isn't it $n_r$ over $n_i$?"
Stephanie: "I do not think so." (softly)
Laura: "Yeah, r over i."
Stephanie: "Is it?"
Olivia: "Yeah, because the slope is y over x."
Stephanie: "Yeah, so it is sin _$_r$ over sin _…"
Olivia: "But y is r."
Laura: "It's r to i."
Olivia: "So it is $n_r$ over $n_i$?"
Stephanie: "No, isn't it still i to r? It's still i to r. Here look…it's r to i and then i to r."
Olivia: "But why is it?"
Stephanie: "Slope is y to x?"
Olivia: "Yeah y is r?"
Stephanie: "…you move this down…"

At this point they were distracted by the other students finding the minimum and maximum values of the slope. On Stephanie's lab report, she ultimately said it represents $n_r$ over $n_i$.

Unfortunately Stephanie was correct the first time, but could not convince her lab partners. Ultimately, she was convinced by them to change her answer to theirs. Max, Ken, and Jerry all had different answers. Max just had: "1.46 = $n_i$" (This is correct but not very descriptive). Ken had: "it represents the refraction index of the plastic cube" (This is correct). Jerry did not describe what it represented.

The students were asked why they graphed rSin vs. iSin instead of iSin vs. rSin. I was expecting students to say that the slope of this graph is the reciprocal of index of refraction of the plastic. However, when the students were interpreting the slope of the inverted graph, iSin vs. rSin, Ken attempted to explain that "iSin vs. rSin represents that if the incident factor [index of refraction] is "one" like air, then this slope (.702) is the factor [index of refraction] for this cube." This interpretation is correct and demonstrates a good understanding of the lab. However, most of the other students either copied his response or their answers did not make much sense. For example, Lauren wrote: "we want refracted/incident, we know that air (_$_r$) is 1 already & w/ iSin vs. rSin, it is false." In this lab we really wanted to know $n_i$, and if $n_r$ is one $n_i$ is the slope of the graph of rSin vs. iSin. _$_i$ is not 1, and I do not know what she means when she says that it is false with iSin vs. rSin. I believe she was trying to interpret what Ken said, but failed to explain it as clearly as he did.

### Pendulum and Energy Conservation (Eastwood High)

This lab had the students examine the conservation of energy by comparing the release height of a cylindrical pendulum bob with its velocity at the bottom of the swing. The students had already been introduced to the concepts of potential energy and kinetic energy. They had solved problems in class using the formulas:

$$PE = mgh$$

$$KE = \frac{1}{2}mv^2$$

where:
PE is the gravitational potential energy
KE is the kinetic potential energy
m is the mass of the object
g is the acceleration constant of gravity
h is the height the object is above the reference height
v is the velocity of the bob

The day before the lab students did a different pendulum lab exploring the relationship between length and period. The students were introduced to the lab, but were not given detailed instructions on how to set up the lab. It was left up to them to set up the lab and to make a prediction of the results they would get.

Once the lab was set up, the students were to have measured the height of the bob at the release point from the table and measured the height of the bob at the bottom of swing from the table. They were to measure the velocity of the bob indirectly with a photogate. A computer attached to the photogate recorded the amount of time the pendulum bob blocked the laser of the photogate. From this blocked time the students could calculate the velocity by dividing the distance it traveled, the diameter of the bob, by the blocked time. After the students recorded the blocked time and a number of release heights, they were to calculate the velocity at the bottom and graph it against the release height. They were then asked a series of questions asking them to analyze their data. After they turned in their lab reports, about two weeks later, I returned to interview them about the lab.

The students had many challenges with this lab. The students did not know how to predict the relationship between speed and release height before the lab. They were confused about what data to collect: both what height they needed to measure and what blocked times they needed to record. The students had a hard time figuring out how to get speed from the blocked time. And when they finally graphed the data they matched the data with a line and ignored the data that did not fit.

The first challenge students faced was predicting the relationship between speed and release height before the lab started. Mica said, "I have no idea." Amy responded, "Wait, what was the formula we used yesterday?" Amy was referring to previous pendulum lab where they changed length and measured period. The students never referred back to the formulas they learned from the class discussion of the conservation of energy. I believe this indicated little understanding of the significance of the equations they had previously used in class for kinetic and potential energy and/or the energy model in physics.

The second challenge the students faced was figuring out what data to collect: both what height they needed to measure and what blocked times they needed to record. As they were setting up the lab, Mica read the instructions that said: "Record the heights and the times through the photogate" and asked, "How do you record the heights? Is it heights like (she lifted the pendulum bob and gestured to the distance from the floor to the bob)?" I responded that was something they had to decide. Mica responded by gasping and looking lost. She said, "Oookay. Alright. Am I allowed to ask [the teacher]?" At one point Amy suggested they measure the length of the string, but Mica did not think this was correct. Later on in the lab as they continued to struggle, I eventually explained the height they needed to measure, but did not mention they needed to subtract the height of the bob at the bottom of the swing. They never independently realized they needed to subtract this height. Ultimately after being told they needed to subtract the height at the bottom, they incorrectly subtracted the height of the photogate instead of the height of the bob at the bottom of the swing.

The third challenge involved units. Mica struggle with measuring the diameter of the pendulum bob. She measured it to be 1 and 7/8th *millimeters* when it was actually centimeters. Later on in the lab she again measured the height of the bob as 21 millimeters. Amy challenged her on this, saying, "No, it is centimeters." Mica disagreed referring to the ruler, "No, it's millimeters and inches. We can convert it later." Amy was not confident enough to challenge her on this. Later on she doubted herself and asked another student who said it was centimeters.

Their fourth challenge was with the blocked time. Once they set up the computer and released the bob the computer would report the blocked times each time the bob passed through the photogate. The first time they collected the data, Amy complained, "I do not have graph." And Mica echoed the problem to me, "We do not have a graph." I explained you do not need a graph, you just need the height and the blocked time. Then they asked if they needed more than one blocked time for each height. They did not understand they only needed one time nor did they understand that they could and should ignore the extra times the computer recorded on the other swings.

Their fifth challenge was with calculating the speed. They did not know how to calculate the speed, and ended up asking me to explain it to them. Once they understood how to calculate speed, they did not know how to do it automatically for all the data with Fathom by creating a calculated column. They had had instruction in the use of Fathom at the beginning of the year, but they either did not cover this or did not remember how to do it.

Their sixth challenge was interpreting their data. At first they graphed blocked time vs. release height, and they thought they were done. They had yet to graph speed vs. release height or find a function which fit their data. The release height they had on the graph was also incorrect because they had not yet subtracted the height of the bob at the bottom of the swing. When asked to find a function that fit the points, Mica immediately suggested a line. Because of the error with the release height and the limited range of the data they took, the data did look like it had a linear relationship. The next day the teacher guided them through the equations of PE and KE to find the expected relationship between velocity and release height:

$$v = \sqrt{2gh}$$

The students, with the help of another student, went back and fit a square root function to their data after correcting the release height. They found the best fit function to be:

$$speed = 35.6 * height^{0.52}$$

During my interviews with the students afterwards they revealed more challenges with the lab. They reported that the lab was difficult because they did not know what they needed to do. In past science labs, the teacher usually demonstrated what they needed to do to collect the data. To make matters worse, the instructions were confusing.

In their lab report, Amy and Mica claimed their results showed the conservation of energy. Amy and Mica claimed: "The weight or the length of the string does not matter, it is only the height at which it is released at. Therefore that proves the conservation of energy." They did not refer back to the formula for potential and kinetic energy or the relationship they found between speed and release height. They had found a relationship between speed and release height, but they did not refer to it in their argument, and when later asked to compare their equation to the one derived from the kinetic and potential energy equations, they could not or would not say it was the same or different. They also claimed, "Our results are not fully accurate because the photogate is not accurate and plus there are other distractions in our lab. To improve more accuracy we can either make some type of computer simulation to get the right results and accurate data." When asked to explain why she thought the photogate was not accurate, Mica, could not say why she thought so. She also could not come up with any other distractions other than wind from passing students.

I was only able to record two students, Mica and Amy, because the students at Eastwood High work in two separate lab groups. The other group encountered similar problems from my observations. In their lab report they reported the derived equation, but claimed: "The model [$speed = 4 * height^{0.57}$] of the graph isn't really accurate because there is some error with our data. For example, our function can vary and still be within the right answers. We were able to come up with a range that the variations can occur and still be right." They did not explicitly compare their formula to the expected relationship. They also did not discuss error and when prompted to compare their results to the expected relationship, they could not or would not say their data and mathematical model was consistent with the expected relationship from the conservation of energy model.

### Mars Needs Rubber (Eastwood High)

The objective of this lab was to launch a rubber band and hit a target, "Mars". The students had recently done a lab measuring how much they needed to sketch a rubber band to get it to touch the ceiling. The students had studied projectile motion several weeks prior to this lab. The students were told to find the distances a rubber band travels at 3 different launch angles and various amounts of stretch. They would then be told how far Mars was, and they would have to choose a launch angle and stretch modeled from their data in order to strike the target. The Mars target was approximately 75 cm in diameter.

The students had some success with this lab, but they did not do what I expected. One lab group was successful in hitting Mars, and the other was not. The students had little trouble collecting data for this lab. They also had little trouble graphing their results. I expected them to model their data with a function, but they did not. When it came time to choose a support value and stretch, both groups choose values of the data point closest to Mars's distance of 5 m. Amy and Mica chose a support value and stretch that they tested had a range of 5.1 m. They successfully hit Mars with their rubber band on the 1st try. Jo and Daphne also just chose setup and stretch values from a data point that gave a range close to 5 meters, but they did not hit Mars with any of their 3 tries. They blamed this on losing the rubber band with which they had taken all of their data.

Both groups displayed some awareness of the confounding variables in their tests, but little awareness of how to gauge the variability in their data. Based on their actions, Daphne and Jo were aware of some of the confounding variables, such as variations between rubber bands and the difference between stretched rubber bands and new rubber bands. Mica and Amy did not report on their lab pages any other values that had effect on the range other than angle of the ruler and the stretch distance. However, Mica did instruct Daphne and Jo to pre-stretch their rubber band to get better results when they were trying to hit Mars. Neither lab group took multiple data points at the same support value and stretch distance to gauge the variability in their data. The lab did not explicitly ask them to do this, but it would have been useful for them to know given their task of hitting Mars.

The students did not attempt to explain their data with theoretical models. The initial velocity could have been estimated based on the elastic energy stored in the rubber bands. The range could have then been modeled based on the equations of projectile motion. The lab did not ask them to do this, and no one in the research groups independently thought to compare these results to the theoretical models. Therefore, this lab was not a good example of integrating math and physics with data analysis. However, the lab did reveal a little bit about the students' thinking about how to control for confounding factors and deal with variability in their data.

### Constant Acceleration on a Ramp (Eastwood High)

The purpose of this activity is to see if the acceleration of a rolling tennis ball down a ramp is constant. In this activity students only analyzed the data that I collected beforehand. I set up a ramp at a fairly shallow angle. I released a tennis ball from various distances and measured the time it took the tennis ball to pass through a photogate at the bottom of the ramp. We provided the students with a Fathom file with all of the data in it as well as a picture of the experimental setup. The students had studied the equations of motion several months prior to this lab. This lab was done after school with only the research students. A full copy of the lab and data the students got is included in the Appendix B.

Unfortunately, due to the last minute scheduling change at the school, many of the students could only stay for a small part of the lab. Amy had to leave immediately, Ben and Jo stayed for about a half an hour, Daphne stayed for an hour, and only Mica stayed for the whole lab.

The students had a lot of challenges with the lab. Initially the students did not know what kinematics equations applied to the situation. They spent 6 minutes looking in their notes and they had made little progress before I helped them get started. After 15 minutes of my coaching they settled on using the relationship below:

$$v_f^{\ 2} = v_i^{\ 2} + 2a\Delta x$$

They concluded that they could calculate the final velocity (the speed_at_bottom) and they knew the distance down the ramp, and that initial velocity was zero. So, I suggested they look at the data because: "they do not want to do a problem in the textbook, you have to look at the data. So instead of doing it for one point you want to do it with more than one point." And Mica responded, "But that's the easy way of it though!"

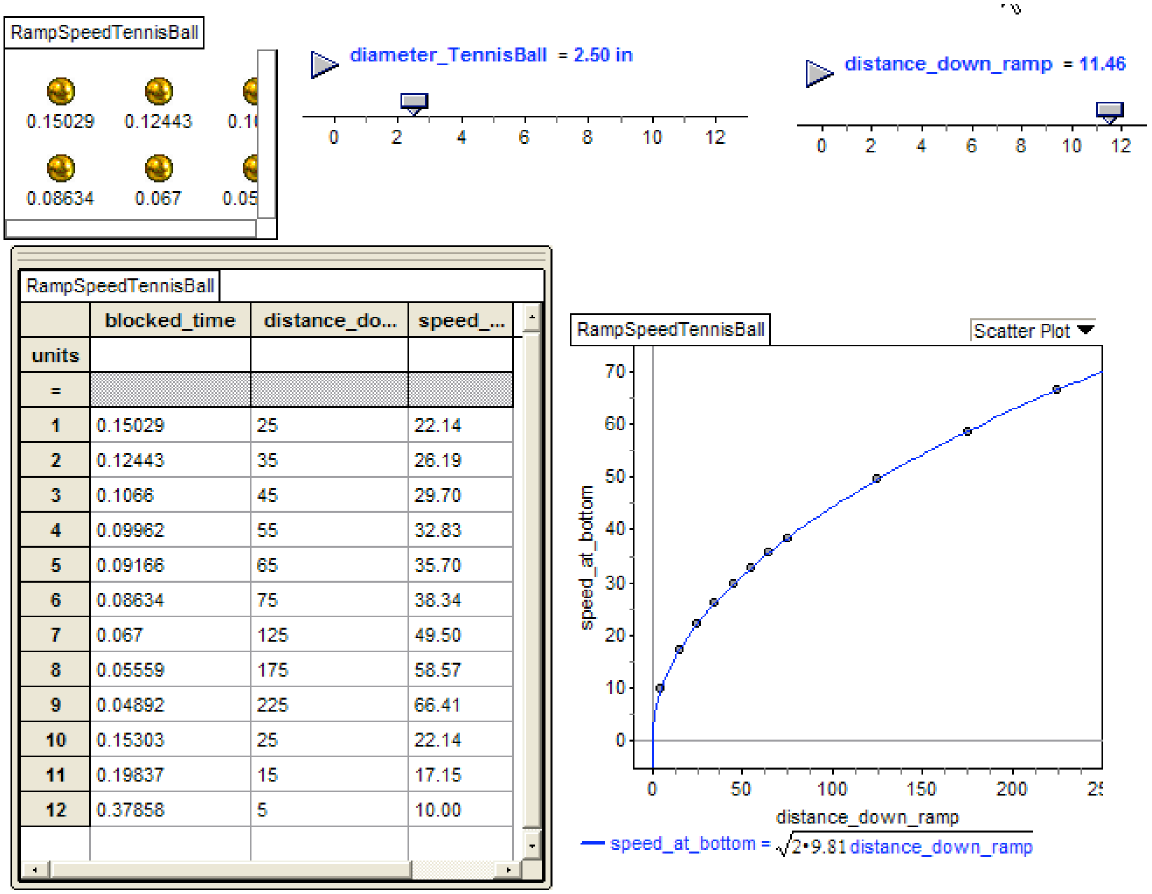
After that I tried to get them to say what relationship they would predict between the data they had. Mica suggested it would be exponential, but would not explain what she meant. So I said, "if you had to say speed_at_bottom equals something what would you say?" When no one responded in 15 seconds, I had them calculate the speed_at_bottom from blocked time and look at the graphs. Mica directed Ben to first graph distance_down_ramp against blocked_time because "it is a time." Eventually they graphed speed_at_bottom versus distance_down_ramp, and Jo recognized it as the square root function. They flipped the axes and looked at the distance_down_ramp vs. speed_at_bottom. The graph looked like a parabola. I asked them what they would expect the relationship to look like. Mica responded: "As distance increases, so does the speed, because the more speed it can accumulate." I responded, "That's the first step, that's the qualitative understanding. But what is the quantitative relationship the theory predicts? Does one of the formulas up there (referring to the equations of motion posted on the wall of their class) predict the relationship you see?" Jo said, "yes, the fourth one does because _x is the distance_down_ramp." This process continued until we plotted a function that fitted the data with the equation shown below:

$$\Delta x = \frac{v_f^{\,2}}{2a}$$

While answering the lab questions, Mica again demonstrated her qualitative understanding of the physical situation with her response to the question: Why do you think the blocked_time goes down as the distance_down_ramp increases? She responded, "…the farther away from the bottom where the photogate thing is, it increases or it accumulates more speed over time, and so like so as it increases the velocity it goes to the photogate, it's like faster so it's like less time that the photogate is blocked." Mica had difficulty with the more quantitative understanding however. She thought the data indicated that the acceleration is constant, but she said she did not know why. And they understood the concept minimum and maximum values of acceleration, but they did not link the uncertainty or error in the data to the process of finding these values.

The context of this lab may have significantly impacted the students' success with this lab. Ben told me later that everyone felt nervous being videotaped and did not want to make a mistake. It was a lot different than the way they may have behaved in a normal physics lab. It had also been several months since the students had studied the material, and it was after school. The students did not know what topic the lab was going to be on before, so they were not able to review prior to the activity. They also did not have a chance to collect the data themselves.

The problem may not have been all context based, however, given the results Jo, Daphne, Ben, and Amy later sent me after they attempted to finish analyzing the lab on their own. Their analysis (a sheet shot of Fathom shown in Figure 4) indicates that they had a lot of challenges analyzing the data. Figure 5 shows what I expected the students to do. They were given the blocked_time and distance_down_ramp, and asked to



RampSpeedTennisBall

| | blocked_time | distance_do... | speed_... |
|---|---|---|---|
| units | | | |
| = | | | |
| 1 | 0.15029 | 25 | 22.14 |
| 2 | 0.12443 | 35 | 26.19 |
| 3 | 0.1066 | 45 | 29.70 |
| 4 | 0.09962 | 55 | 32.83 |
| 5 | 0.09166 | 65 | 35.70 |
| 6 | 0.08634 | 75 | 38.34 |
| 7 | 0.067 | 125 | 49.50 |
| 8 | 0.05559 | 175 | 58.57 |
| 9 | 0.04892 | 225 | 66.41 |
| 10 | 0.15303 | 25 | 22.14 |
| 11 | 0.19837 | 15 | 17.15 |
| 12 | 0.37858 | 5 | 10.00 |

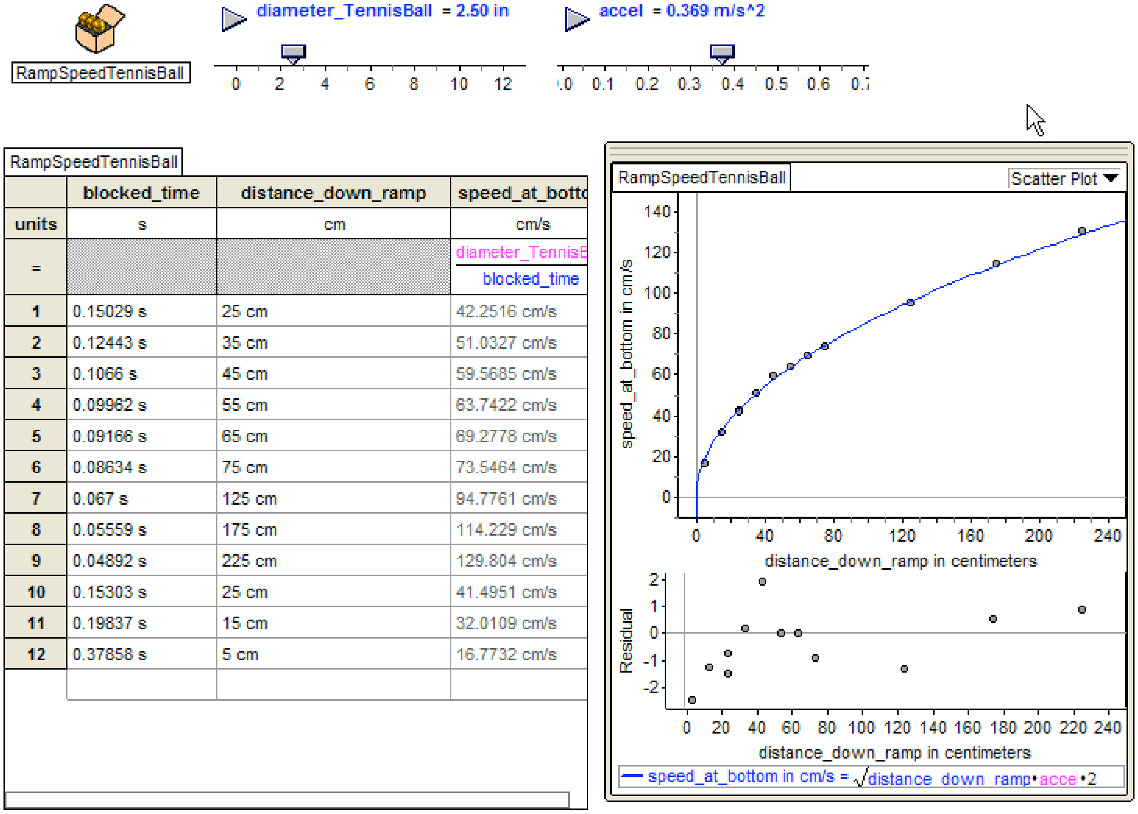
**Figure 4.** Students' analysis of the data from the Constant Acceleration on a Ramp lab.

calculate the speed_at_bottom. You can see from the numbers in the speed_at_bottom column that they did not divide the diameter of the tennis ball by the blocked time as they should have done. Instead they seem to have calculated the speed by plugging the numbers into the equation:

$$speed\_at\_bottom = \sqrt{2*9.81*DISTANCE\_down\_ramp}$$

This is the equation they used in the Pendulum and Energy Conservation lab, but it is not the correct equation here. By using the same function they used to calculate the speed_at_bottom in the graph, they fit all of their points exactly. In this case their graph just illustrates the equation they used to calculate the speed_at_bottom in the first place. They cannot use it to estimate the acceleration the Tennis Ball experienced going down the ramp, as the analysis in Figure 5 demonstrates. The students used g (9.81) in their equation, when they should have used a slider. A slider is an object in Fathom that allows students to modify a parameter by sliding the block with the arrow along the number line until the function fits their data. In Figure 5, the acceleration is represented by the slider called “accel”. The students' analysis indicates that: they did not understand how to calculate the speed_at_bottom, they had difficulty using Fathom, and they probably did not understand the purpose of the activity.

Notice also that the students removed the units in their analysis. This may indicate trouble with units themselves or just the Fathom instructions they received from their instructor. They were using an alpha version of Fathom, so units have not been working consistently.



diameter_TennisBall = 2.50 in

accel = 0.369 m/s^2

RampSpeedTennisBall

|  | blocked_time | distance_down_ramp | speed_at_botto |
|---|---|---|---|
| units | s | cm | cm/s |
| = |  |  | diameter_TennisB<br>blocked_time |
| 1 | 0.15029 s | 25 cm | 42.2516 cm/s |
| 2 | 0.12443 s | 35 cm | 51.0327 cm/s |
| 3 | 0.1066 s | 45 cm | 59.5685 cm/s |
| 4 | 0.09962 s | 55 cm | 63.7422 cm/s |
| 5 | 0.09166 s | 65 cm | 69.2778 cm/s |
| 6 | 0.08634 s | 75 cm | 73.5464 cm/s |
| 7 | 0.067 s | 125 cm | 94.7761 cm/s |
| 8 | 0.05559 s | 175 cm | 114.229 cm/s |
| 9 | 0.04892 s | 225 cm | 129.804 cm/s |
| 10 | 0.15303 s | 25 cm | 41.4951 cm/s |
| 11 | 0.19837 s | 15 cm | 32.0109 cm/s |
| 12 | 0.37858 s | 5 cm | 16.7732 cm/s |

speed_at_bottom in cm/s = √distance_down_ramp•accel•2

Figure 5. The analysis I expected students to do with the data from Constant Acceleration on a Ramp lab.

# Findings

Overall, the students did have a lot of challenges integrating math and physics with data-analysis-rich labs. Despite all the challenges with the labs, all but one of the students thought that the labs were more useful than a lecture with demonstrations on the same topic. Their answers may have been influenced because they wanted to please me, the interviewer. I was still surprised that they preferred the labs, especially with all the challenges they had with the labs. The lone dissenter, Max, felt a lecture might be more useful than doing the data-analysis-rich labs. Ken thought the labs overall were useful, but he expressed dissatisfaction with the Farley Effect lab. He never understood the lab, and thought it should be modified. The other students said the labs helped them remember the material, and helped make the material they were learning more meaningful. Six of the students mentioned that more discussion before and after the lab in class would have helped them to understand the material and the labs a lot better. Perhaps more discussion before and after the labs would have helped the students develop the necessary perspective, equation fluency, and data analysis skills over time. Regardless of the students' opinions about the usefulness of the labs, they did demonstrate that students' success does depend on the context, their equation fluency, their data analysis skills, and their perspective.

The context of the research may have exacerbated the challenges the students had with integrating math and physics through data analysis above and beyond the challenges they would normally face in their physics class. As Ben mentioned to me, videotaping the students made them more nervous and may have inhibited their contributions for fear of making mistakes. The labs themselves were added into the physics teacher's curriculum rather than integrated by the teachers. An attempt was made to integrate the labs into the teachers' current curriculum, but there were some problems. For example, the notation for voltage was different in the Farley Effect Lab from the Oceanbeach teacher's. And the students did not get a chance to review refraction before they were asked to do the Snell's Law lab. At Eastwood High, the Constant Acceleration on a Ramp lab in particular did not have anything to do with the current topics they were studying. The labs were often not part of a learning cycle of exploration, term introduction, and concept application as Lawson suggests science curriculum should be organized. There often was too little time to properly discuss the phenomenon and hypotheses as a class before the students were asked to analyze the quantitative data. The labs themselves, for the most part, had not been tested with real students, and the students identified the language and type of questions the labs asked as unfamiliar and hard to understand at times.

The teachers' pedagogical styles and experience also played a part in the context in which the labs took place. Both teachers used a more traditional style of instruction with lectures and demonstrations rather than focus their instruction explicitly through modeling. Hake (1998) has shown significant learning benefits of a modeling approach to physics instruction. And without a good grasp of the underlying models, it is often difficult to develop much equation fluency. For example, the majority of the Oceanbeach students misapplied a resistance formula to their results on the Ohm's Law lab. At Eastwood High the students had a very difficult time applying the kinematics equations to the Constant Acceleration lab and a difficult time applying the energy equations to the Pendulum and Conservation of Energy lab. The students at Eastwood High's had low FCI scores, indicating they had limited access to the concepts of Newtonian mechanics. This helps explain why they had such a hard time with the constant acceleration lab. Perhaps a modeling-focused instruction style would have helped the students understand the need to put the equations they learned into a cohesive model. This would help the students apply the equations to the physical situations they encountered in the labs.

The students also were often confused by their past experiences in class. The students frequently had a hard time because they applied old equations or relationships to new situations. For example, in the Farley Effect lab, Ken, Max, and Stephanie did not understand the new relationship between voltage and current. They kept trying to apply the directly proportional relationship between voltage and current they found in the Ohm's Law lab to the circuit in the Farley Effect lab. They never developed a new model for the new situation, so they never were able to apply their prior experience with Ohm's law effectively. Jo, Daphne, Ben, and Amy also incorrectly applied the equation from the Pendulum and Energy Conservation lab to the Constant Acceleration on a Ramp lab. And in the pendulum lab, Amy suggested they measure the length of the string as the height, confusing the lab with the one they did before.

Many of the students also encountered challenges due to their lack of equation fluency. For example, in the Snell's Law lab Olivia and Laura convinced Stephanie that the slope on the graph of rSin vs. iSin was not $n_i$ / $n_r$ when it actually was. If any of them could have simplified Snell's Law for (sin _$_r$) they could have come up with the correct interpretation of the slope. The equation of Snell's Law ($n_i \sin\theta_i = n_r \sin\theta_r$), where the variables are inside sine function, may have contributed to their confusion. This confusion indicates a misunderstanding of slope. The Eastwood High students also had difficulty with equation fluency, specifically when they were predicting the relationship between the speed_at_bottom and distance_down_ramp. They were given the equation (in different notation that I explained) and they were unable to simplify and translate it into the terms used in the lab. This also seems to indicate that the students have challenges simplifying equations in physics and relating those equations to the data.

Some problems with equation fluency could be the result of uncertainty about the underlying model. The Eastwood High students had solved problems with the equations for kinetic and potential energy, but they could not use them to predict a relationship between velocity and height in the Pendulum and Energy Conservation lab until after instructed to do so. Even then, Amy and Mica did not include this theoretical relationship in their lab report. In their lab report, Jo, Ben, and Daphne did find the relationship between velocity and height from the kinetic and potential energy equations. Jo, Ben, and Daphne still did not compare it to their own function they found from their data. It seems both during and after the lab, they did not realize these equations applied to the physical situation they were exploring. Perhaps if they had a firmer grasp of the underlying model they would have been able to apply the formulas.

Several students also had challenges because they had limited data analysis skills. In number of different places the students had challenges with units and measurement. They also had challenges processing their data, and using the equipment.

Several students had challenges with units and measurement. For example, all but one of the students at Oceanbeach High could not understand how the multimeter settings for measuring current just showed the same current with highest precision in different units. And Ken forgot to consider units when he found the slope of the voltage vs. current, and failed to catch his mistake even though he gave an example that predicted a current 26 A (a very large amount of current). Mica mistakenly thought the ruler measured millimeters instead of centimeters, and Amy left that mistake unchallenged.

Students also had trouble collecting and processing their data into the variables they wanted to examine. In the Pendulum and Energy Conservation lab, Mica and Amy had a hard time measuring the height of the pendulum bob and did not understand they had to subtract the height of the pendulum bob at the bottom of the swing. This made it a lot easier to misinterpret their data as having linear relationship instead of a radical or square root relationship. In the same lab, the students were unable to calculate speed from the blocked time of a photogate without assistance. The students had the same problem in the Constant Acceleration on a Ramp lab when they had to calculate the speed_at_bottom from the tennis ball diameter and the blocked time.

The students had challenges understanding how some of their measuring equipment worked and how Fathom worked. The students at Oceanbeach High in the Farley Effect lab did not know that a multimeter measures the voltage drop in a circuit between the two points it touches. This misunderstanding of the voltage they were measuring prevented them from being able to interpret the data successfully. Also at Oceanbeach High, the confusion over interpreting the inverted graphs in the Ohm's Law lab was an effect of the autoscaling feature of Fathom. Even if the students were not using Fathom, understanding the impact scale can have on graphs is an important data analysis skill. Also in the Ohm's Law lab, Stephanie blamed human error when the data they had collected contradicted her inappropriate application of a formula for the resistance. Stephanie also blamed human error even though the results for each of the resistors tested were very clean and showed clear relationships. In the Pendulum and Energy Conservation lab, in her lab report Mica blamed the equipment, claiming the photogate was inaccurate without any obvious reason or proof. There was no obvious reason to doubt the accuracy of that data. Perhaps Mica just did not understand how it worked and so it was easy to blame any error on the photogate.

In a few cases a student's perspective also limited their ability to integrate math and physics. Six of the students believe that in physics, mathematical formulas provide ways to get numerical answers to problems more than they express meaningful relationships among variables. In particular, Mica's VASS survey she indicated that she thought mathematical formulas in physics are nearly only used to provide ways to get numerical answers to problems rather than express meaningful relationships among variables. And during the Constant Acceleration on a Ramp lab she understood the lab qualitatively, but did not or could not use the equations to express the relation between speed_at_bottom and distance_down_ramp. She also commented using formulas to find the answer to a textbook problem was easy with the kinematics equations, but groaned when she had to apply it to the data. The other students at Eastwood High also had a hard time using equations to express meaningful relationships instead of a way to get numerical answers. In the same lab, they ended up plugging the data into an equation and graphing their results instead of using data to determine the acceleration parameter by fitting the right relationship to the data.

# Implications

The data-analysis-rich labs were very challenging for the students in this study, especially the students at Eastwood High. The thinking required to answer many of our questions and perform the required analysis was clearly beyond many of the students' abilities. We need to better prepare our students to face these challenges, or we need to give them easier tasks. Giving them easier tasks would do the students a disservice. So we need a way to better prepare our students to face the challenge of integrating math and physics with data-analysis-rich labs.

The research indicates that many students do not have a firm grasp of equation fluency or data analysis skills. Their perspective is also often limiting. We clearly need to be more explicit about how math is different with physics equations. We also need to be more explicit about the importance of the underlying models that give the equations meaning. However, if a student is not yet ready intellectually to grasp proportional reasoning, it may not be enough. Similarly with data analysis, many students would benefit if we are more explicit about data analysis skills, such as those dealing with measurement, with units, with graph interpretation, and with error. Again, if the students have not developed the practice or ability to use combinatorial thinking and to identify and to control variables, the explicit instruction may not be useful. We need to find ways to accelerate our students' intellectual development. We need to work together with other science teachers and math teachers to develop our students' thinking. Perhaps being explicit about the underlying models will allow students to develop their thinking abilities. Modeling-focused instruction has been shown to increase students' conceptual understanding of physics. Lawson suggests incorporating the labs we do in physics into a learning cycle of exploration, term introduction, and application is the most effective way to teach science. Perhaps this is the key to accelerating our student's intellectual development.

The students' attitudes about physics seemed to impact their engagement with the labs as well. Open conversations about student and "expert" attitudes may help students become aware of and possibly adjust some of their beliefs that may be preventing them from appreciating the full meaning of what they are learning.

In particular, we must address the attitude about equations as a means to find numerical solutions, rather than a meaningful expression of the relationships between variables. Perhaps the homework and the tests students take in physics are placing too much emphasis on finding the correct numerical answer. We need take a close look at what messages our assessments are sending to the students about what is really important. Having students find the correct answer does not necessarily help them understand the underlying model. If a student learns how to solve a particular type of problem and repeats that process on a test, it does not necessarily mean they understand the physics. The research students had a hard time applying the equations to show relationships instead of using them to solve for a numerical answer, but many of them still got an A or A- in the course.

The context of Oceanbeach High and Eastwood High was significantly different. While all of the students involved in the research were above average, the students at Oceanbeach High on average were better prepared for the challenge of integrating math and physics. You must get to know your students and their abilities because one school or one class might be significantly different than another class. Any lab or curriculum you make must meet them where they are or just a bit beyond to push their learning. However, if you do a lab where most of the students are not successful, it is critical to follow up on the lab in class. For example, most of the students at Eastwood High had a very difficult time with the Pendulum and Energy Conservation lab, but based on my interviews with them and their lab reports, it seems they did not get enough post review of the lab to capitalize on all the learning possible from the experience.

Follow-up is important, but so is preparation. In many of the labs, the students did not have a good grasp of the purpose of the lab or the underlying model, so the data-analysis-rich lab was very difficult for them. For example, a class discussion about the model underlying the Farley Effect lab may have clarified how the situation was different than the Ohm's Law lab, so students would not have confused themselves trying to find the same relationships they did for the Ohm's Law lab. How to lead this discussion is a good question. I think it would be important for students to discuss the model and come up with some hypotheses to test before having them do a data-analysis-rich lab. This might avoid some of the problems I saw with the labs during the study.

These labs were an attempt to close some important gaps in our students' educations. Integrating math and physics with data highlights the need for students to develop intellectually. If we get caught up focusing on our own disciplines standards, we may lose sight of these important educational goals. The math teacher might think it is the physics and science teachers' jobs to teach the students how to deal with all the issues surrounding data analysis. And the physics and science teachers might think the math teachers have developed the students' equation fluency. And neither teacher is asking the students to think at a hypothetical-deductive level because there are too many standards to teach already. But when students analyze data, they get a chance to integrate math and physics and practice a set of skills critical to their intellectual development. They get to see math applied in real contexts and they get to use their mathematical abilities to understand the relationships in physics at a much deeper level. By identifying the challenges facing students trying to integrate math and physics, I have realized the value of spending more time addressing these skills and abilities that are being left behind, and I hope it encourages other math and science teachers to do likewise. If we do not, our students will not come close to their full capabilities.

## References

Adkins, Jeff (2004) http://homepage.mac.com/astronomyteacher/dvhs/grapheq.html

Beichner, Robert J. (1994) "Teaching student interpretation of kinematics graphs" Am. 3. Phys. 62 (8), August

Beichner, Robert J. (1996) "The impact of video motion analysis on kinematics graph interpretation skills" Am. J. Phys. Vol 64 No. 10, October

Bowden, G. Dall'Alba, and Martin, E. et. (1992) "Displacement, velocity, and frames of reference: Phenomenographic studies of students' understanding and some implications for teaching and assessment" Am. J. Phys. 60 (3), March

Foerster, Paul A. (1999) *Algebra and Trigonometry: Functions and Applications*. Addison-Wesley

Grayson, Diane J. and McDermott, Lillian C. (1996) “Use of the computer for research on student thinking in physics” Am. J. Phys. 64 (5), May

Hake, R. (1998) Interactive-engagement vs traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses, Am. J. Phys. 66, 64-74

Halloun, Ibrahim (1996) “Views about science and physics achievement.  The VASS Story.” *Proceedings of the International Conference on Undergraduate Physics Education*, American Institute of Physics Press, College Park, MD.

Hestenes, David, Wells, Malcolm, and Swackhamer, Gregg (1992) “Force Concept Inventory” The Physics Teacher 30, March p. 141-151

Hestenes, David (1996) “Modeling Methodology for Physics Teachers” Proceedings of the International Conference on Undergraduate Physics Education (College Park, August)

Lawson, Anton E. (1995) *Science Teaching and the Development of Thinking,* Wadsworth Publishing Company. Belmont, CA

McDermott, Lillian C., Rosenquist, Mark L., and van Zee, Emily H. (1987) “Student difficulties in connecting graphs and physics: Examples from kinematics” Am. J. Phys. 55 (6), June

McDermott, Lillian C. Shaffer, Peter S., and Somers, Mark D. (1994) "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine" Am. J. Phys. 62 (1) January

Redish E. F. (1994) “Implications of cognitive studies for teaching physics,” Am. J. Phys. 62, 796-803

Redish, Edward F., Saul, Jeffery M., Steinberg, Richard N. (1997) “On the effectiveness of active-engagement microcomputer-based laboraties” Am. J. of Phys. 65 (1), January

Swatton, P. (1995a) “Pupil performance in data manipulation and its relationship to the skill of interpretation” Educational Review Vol. 47 Issue 1

Swatton, Peter (1995b) “Can we know whether pupils are passing the ‘fair test’?” Research Papers in Education 10 (1) pp.51-73

Tamir, Pinchas ref. in Shulman, Lee S. (1988) “Teaching Alone, Learning Together: Needed Agendas for the New Reforms” in *Schooling for Tomorrow: Directing Reforms to Issues That Count*. T. J. Sergiovanni and J. H. Moore (Eds.). Boston: Allyn and Bacon

Thornton, Ronald K. Sokoloff and David R. (1987) "Learning motion concepts using real-time microcomputer-based laboratory tools" American Journal of Physics 55, 858-867